Christ Church Grammar School

2018
TEST 4

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Student name SOLUTIONS

Teacher name _____

Time and marks available for this section

Reading Time:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(5 marks)**

The fraction $\frac{5x+11}{(x+1)(x+3)}$ can be re-written as the sum of two fractions.

That is $\frac{5x+11}{(x+1)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+3)}$, where A and B are real numbers.

(a) Calculate the values of A and B. (3 marks)

$$\frac{5x+11}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$\Rightarrow \quad 5x+11 = A(x+3) + B(x+1)$ ✓

If $x=-3$ $\quad -15+11 = 0 + -2B$

$-4 = -2B \Rightarrow B = 2$

If $x=-1$ $\quad -5+11 = 2A + 0$

$6 = 2A \Rightarrow A = 3$

$\therefore A = 3$ and $B = 2$ ✓ ✓

If incorrect calculation check to see if values are correct and therefore 1/3.

can also solve

$5x+11 = Ax+3A+Bx+B$

$\Rightarrow \quad 5 = A+B$

$11 = 3A+B$

and solve simultaneously.

(b) Hence or otherwise integrate the following function:

$$\int \frac{5x+11}{(x+1)(x+3)}\,dx$$ (2 marks)

$= \int \left(\frac{3}{x+1} + \frac{2}{x+3}\right) dx$

$= 3\ln|x+1| + 2\ln|x+3| + C$

✓ ✓

F.T if error from (a).

Question 2 **(7 marks)**

Integrate each of the following:

(a) $\int \cos^2\left(\frac{x}{4}\right) dx$ (3 marks)

$= \int \left(\frac{1}{2}\cos\left(\frac{x}{2}\right) + \frac{1}{2}\right) dx$

or $\frac{1}{2}\int \left(\cos\left(\frac{x}{2}\right) + 1\right) dx$

$= \frac{1}{2}\left[\frac{\sin\left(\frac{x}{2}\right)}{1/2} + x\right] + c$ ✓

$= \sin\left(\frac{x}{2}\right) + \frac{x}{2} + c$ ✓

$\cos^2\left(\frac{x}{4}\right) = \frac{\cos 2\left(\frac{x}{4}\right)}{2} + \frac{1}{2}$

$= \frac{1}{2}\cos\left(\frac{x}{2}\right) + \frac{1}{2}$ ✓

(b) $\int \frac{x^3}{x^2-1} dx$ (4 marks)

$= \int \left(x + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}\right) dx$

$= \frac{x^2}{2} + \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + c$

✓ ✓

$x^2-1 \overline{)\, x^3 + 0x^2 + 0x + 0}$ $\quad x$

$-(x^3 - 0x^2 - x)$

$0x^2 + x$

$\therefore x + \frac{x}{x^2-1}$ ✓

and $\frac{x}{x^2-1} = \frac{x}{(x-1)(x+1)}$ (not needed.)

$= \frac{A}{x-1} + \frac{B}{x+1}$

$\Rightarrow x = A(x+1) + B(x-1)$

$x=1 \quad 1 = 2A \Rightarrow A = \frac{1}{2}$

$x=-1 \quad -1 = -2B$ ✓

$\therefore B = \frac{1}{2}$

can still get mark for A and B if long division wrong.

* also accept $\int \left(x + \frac{x}{x^2-1}\right) dx = \frac{x^2}{2} + \frac{1}{2}\ln|x^2-1| + c$ ✓

3rd mark if don't do fractions

If error in A and/or B calculation can still get marks for integration

Note: -1 mark for no $+c$ from 1b, 2a or 2c.

See next page

Question 3 **(4 marks)**

Calculate the exact area ofthe following integral using the substitution of $u = 4 - x^2$, if required.

$$\int_0^2 xe^{4-x^2}dx.$$

$u = 4 - x^2$

$\frac{du}{dx} = -2x$ ✓

$\frac{du}{-2x} = dx$

$x = 0,\ u = 4$

$x = 2,\ u = 0$ ✓

$= \int_4^0 -\frac{1}{2} e^u \, du$

$= \frac{1}{2}\int_0^4 e^u \, du$

$= \frac{1}{2}\left[e^u\right]_0^4$ ✓

$= \frac{1}{2}\left(e^4 - e^0\right)$

$= \frac{1}{2}(e^4 - 1)$ units^2 ✓

accept $\left(\frac{1}{2}e^4 - \frac{1}{2}\right)$ units^2

or $\int_0^2 xe^{4-x^2}dx$

$= \left[\frac{e^{4-x^2}}{-2}\right]_0^2$ ✓

$= \left(\frac{e^{4-2^2}}{-2}\right) - \left(\frac{e^{4-0}}{-2}\right)$ ✓

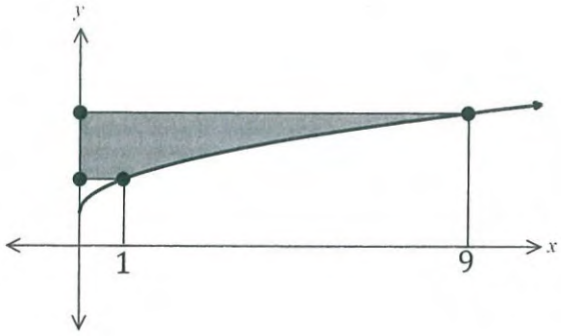
$= \left(\frac{e^0}{-2}\right) - \left(\frac{e^4}{-2}\right)$ ✓

$= \left(\frac{e^4}{2} - \frac{1}{2}\right)$ units^2 or $\frac{1}{2}(e^4 - 1)$ units^2 ✓

See next page

Question 4 **(4 marks)**

Calculate the area trapped between the function $y = \frac{\sqrt{x}+1}{2}$ and the y-axis in the interval $1 \leq x \leq 9$.



If $x=1$, $y = \frac{\sqrt{1}+1}{2} = 1$ ✓

$x=9$ $y = \frac{\sqrt{9}+1}{2} = 2$

$2y-1 = \sqrt{x}$

$(2y-1)^2 = x$ ✓

$A = \int_1^2 (2y-1)^2 \, dy$

$= \left[\frac{(2y-1)^3}{3\times 2}\right]_1^2$ ✓

$= \frac{(2(2)-1)^3}{6} - \frac{(2(1)-1)^3}{6}$

$= \frac{27}{6} - \frac{1}{6}$

$= \frac{26}{6}$

$= \frac{13}{3}$ $(4\frac{1}{3})$ units^2 ✓

See next page

Question 5 **(4 marks)**

Calculate the exact value of the following integral in terms of c.

$$\int_0^c \frac{x}{\sqrt{x^2+4}}\,dx$$

$u = x^2 + 4$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$ ✓

$x = 0,\ u = 4$ ✓

$x = c\ u = c^2 + 4$

$= \int_4^{c^2+4} \frac{1}{2u^{1/2}}\,du$ ✓

$= \frac{1}{2}\int_4^{c^2+4} u^{-1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]_4^{c^2+4}$

$= u^{1/2}\Big]_4^{c^2+4}$

$= \sqrt{c^2+4} - \sqrt{4}$

$= \sqrt{c^2+4} - 2$ ✓

either method accepted.

or $\int_0^c x.(x^2+4)^{-1/2}\,dx$ ✓

$= \frac{x.(x^2+4)^{1/2}}{1/2 \times 2x}\Big]_0^c$ ✓

$= (x^2+4)^{1/2}\Big]_0^c$

$= (c^2+4)^{1/2} - (0^2+4)^{1/2}$ ✓

$= \sqrt{c^2+4} - \sqrt{4}$

$= \sqrt{c^2+4} - 2.$ ✓

See next page

Question 6 **(6 marks)**

Integrate the following function using the substitution $x = 3sin\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}dx$$

$x = 3\sin\theta$

$x^2 = 9\sin^2\theta$

$\frac{dx}{d\theta} = 3\cos\theta$ ✓

$dx = 3\cos\theta\, d\theta$

$= \int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\, d\theta$ ✓

$= \int \frac{9\sin^2\theta}{\sqrt{9(1-\sin^2\theta)}} \cdot 3\cos\theta\, d\theta$

$= \int \frac{9\sin^2\theta}{\sqrt{9\cos^2\theta}} \cdot 3\cos\theta\, d\theta$

$= \int \frac{9\sin^2\theta}{3\cos\theta} \cdot 3\cos\theta\, d\theta$ ✓

$= \int 9\sin^2\theta\, d\theta$

$\sin^2\theta = \frac{1}{2} - \frac{\cos 2\theta}{2}$

$= 9\int \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right) d\theta$ ✓

$= 9\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right) + C$

$= \frac{9\theta}{2} - \frac{9\sin 2\theta}{4} + C$ ✓

$= \frac{9\theta}{2} - \frac{9}{2}\cdot\frac{1}{2}\sin\theta\cos\theta + C$

$= \frac{9\theta}{2} - \frac{3\sin\theta\, 3\cos\theta}{2} + C$

$\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x}{2}\sqrt{9-x^2} + C$ ✓

End of questions

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MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Student name SOLUTIONS

Teacher name _____

Time and marks available for this section

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Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

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To be provided by the candidate

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Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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See next page

Question 7 **(2 marks)**

Integrate the following, give answers to 2 decimal places if necessary.

$$\int_1^2 \left(x^2 + \frac{1}{x}\right)^4 dx$$

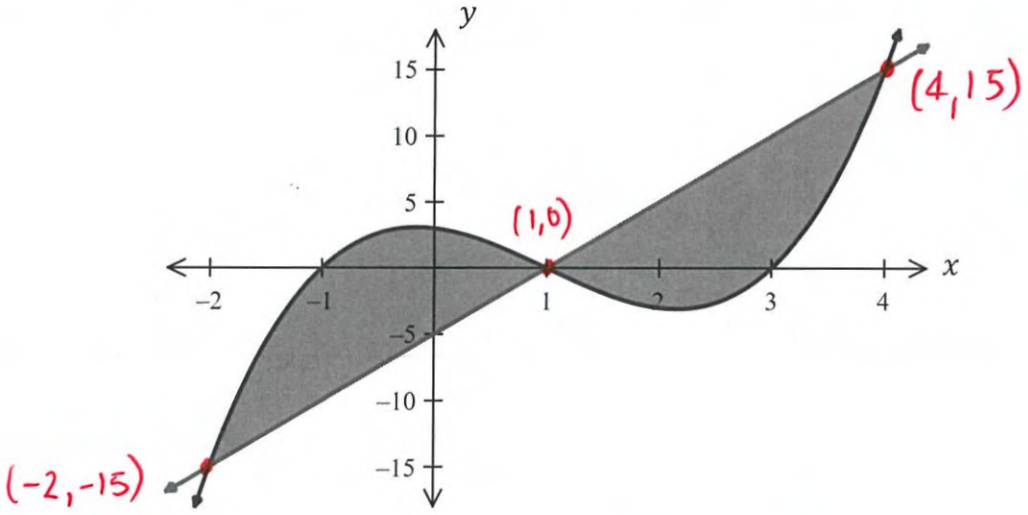
$= 4\ln 2 + \frac{81}{72}$

or ≈ 115.84 (2dp) ✓✓

accept either but -1 if don't round to 2dp.

Question 8 **(3 marks)**

The curve has an equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. Calculate the total shaded area.



✓ mark for both intersection points

$$A = \int_{-2}^{1} \left((x+1)(x-1)(x-3) - (5x-5)\right) dx + \int_{1}^{4} \left((5x-5) - (x+1)(x-1)(x-3)\right) dx$$ ✓

$= 20\frac{1}{4} + 20\frac{1}{4}$

$= 40\frac{1}{2}$ units^2 ✓

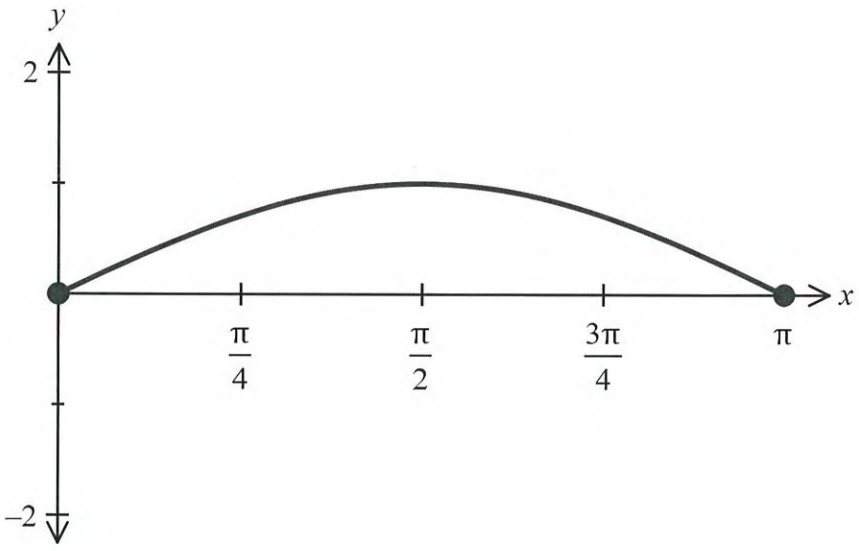
or $A = 2 \times \int_1^4 \left(5x - 5 - (x+1)(x-1)(x-3)\right) dx$

$= 40\frac{1}{2}$ units^2

answer only 2/3

Question 9 **(3 marks)**

Let $f(x) = x\sin(x)$, for $0 \leq x \leq \pi$.



If this shape is rotated about the $x-axis$ through 360°

(a) Write down an expression for this volume of revolution. (2 marks)

$V = \pi\int_0^{\pi} y^2\,dx$ ✓

$= \pi\int_0^{\pi} (x\sin x)^2\,dx$ ✓

2 marks for final expression is also okc.

(b) Calculate the volume. (1 mark)

$V = \dfrac{\pi^2(2\pi^2 - 3)}{12}$

or $\simeq 13.77$ units^3 (2dp) ✓

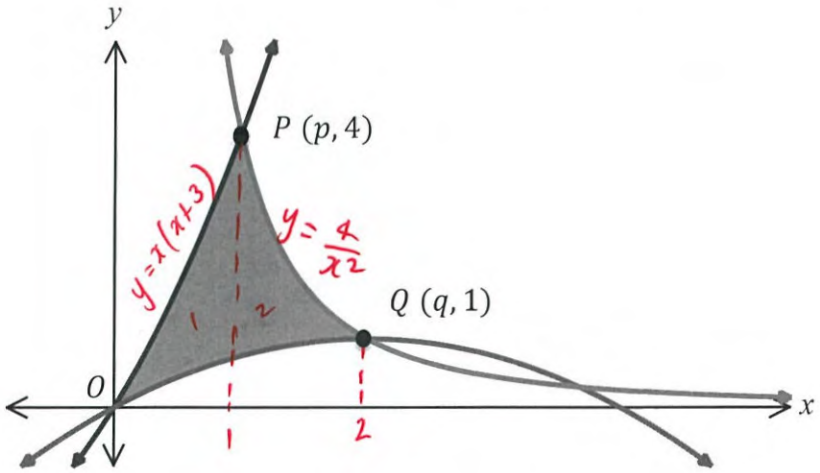
or $\left(\dfrac{\pi^4}{6} - \dfrac{\pi^2}{4}\right)$ units^3

1 mark for any of 3 solutions.

See next page

Question 10 **(7 marks)**

The origin, O and the points P and Q are the vertices of a curved "triangle" which is shaded in the diagram.
The sides lie on the curve with the equations $y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.



(a) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. (2 marks)

$p = 1$ ✓ $q = 2$ ✓ (solve on calculator)

(b) Calculate the shaded area, clearly indicating anything you type into your ClassPad. (5 marks)

area 1: $\int_0^1 \left(x(x+3) - \left(x - \frac{1}{4}x^2\right)\right) dx$ ✓

$= 1\frac{5}{12}$ $(1.41\dot{6})$ units² ✓

area 2: $\int_1^2 \left(\frac{4}{x^2} - \left(x - \frac{1}{4}x^2\right)\right) dx$ ✓

$= 1\frac{1}{12}$ $(1.08\dot{3})$ units² ✓

∴ Shaded area = 2½ units² ✓

End of questions

Additional working space

Question number:_____

Additional working space

Question number:_____